INFORMATION CAPACITIES OF "HYBRID" COMMUNICATION CHANNELS

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INTRODUCTION

The protocol of *entanglement-assisted classical communication* was introduced by Bennett, Shor, Smolin and Thapliyal as a generalization of superdense coding to noisy quantum channels. Important observation: entanglement-assisted communication may be advantageous even for *entanglement-breaking* channels.

We provide further results in this direction by considering two distinguished classes of entanglement-breaking channels, namely *measurements* (quantum-classical) and *preparations* (classical-quantum). For both, called here "hybrid" for brevity, we compute and compare the (unassisted) *classical capacity* C and the *entanglement-assisted classical capacity* C_{ea}.

MOTIVATION AND APPLICATIONS

C, C_{ea} give ultimate measures of information processing performance

- Communication of classical information via quantum channels

- Storage/retrieval of classical information from quantum memory

- Information capacity of quantum sources and detectors

MAIN RESULTS

- *Pure measurement (q-c) channels* demonstrate the maximal gain of entanglement-assistance. Typically $C_{ea}/C \sim -\log \mathtt{SNR}$ for weak signals.

- On the contrary, *preparation (c-q) channels* are essentially characterized by no gain of entanglement-assistance: $C_{ea}/C = 1$.

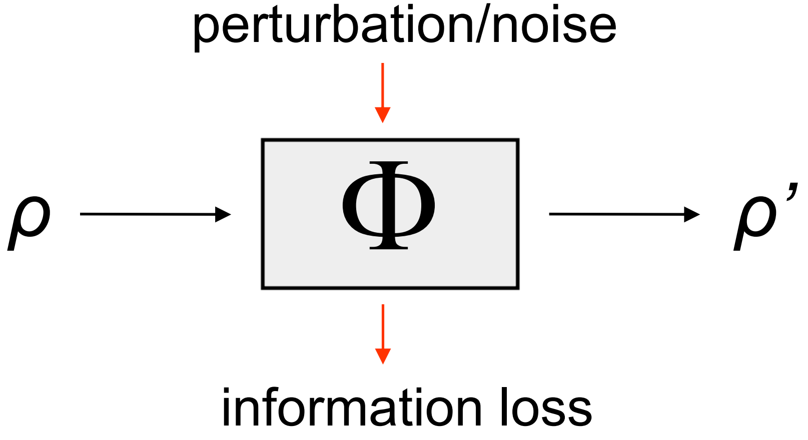
- But the *input constraints* (e.g. energy constraint in continuous variable systems) can restore the gain for c-q channels.

MEASUREMENTS ARE MORE QUANTUM THAN PREPARATIONS

Quantum channel

In general, quantum channel is given by trace-preserving completely positive map $\rho \rightarrow \Phi(\rho)$, transforming input states into output states (*Schroedinger picture*).

The map gives condensed statistical description of the outcome of interaction of the system with its environment.



CLASSES OF CHANNELS

quantum (q-q) channel = CPTP mapping of quantum states: $\rho \longrightarrow \rho' = \Phi[\rho]$.

c-q channel = *state preparation*: $x \to \rho_x$

q-c channel = *measurement*: mapping of quantum states into probability distributions: $\rho \longrightarrow P_\rho$; $P_\rho(y) = \mathrm{Tr}\rho M_y$.
Observable = probability operator-valued measure (POVM):
$M = \{M_y; y \in \mathcal{Y}\}$; $M_y \geq 0$, $\sum_y M_y = I$

q-c-q channel = entanglement-breaking

c-c channel = classical noisy channel

EMBEDDING CLASSICAL INTO QUANTUM

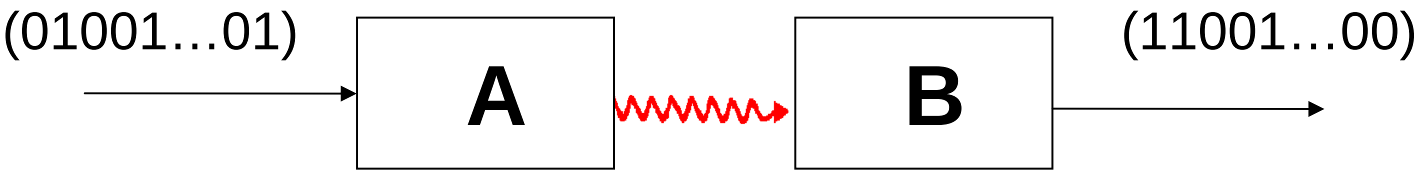
Any "hybrid" channel with discrete input alphabet $\mathcal{X}$ or output alphabet $\mathcal{Y}$ can be regarded as a quantum channel $\rho \to \Phi(\rho)$, which allows us to use the capacity formulas for quantum channels. In the case of *c-q channel* $x \to \rho_x \,(x \in \mathcal{X}\ )$, the quantum channel is

$$\mathcal{P}(\rho) = \sum_{x\in\mathcal{X}} \langle x|\rho|x\rangle \rho_x, \tag{1}$$

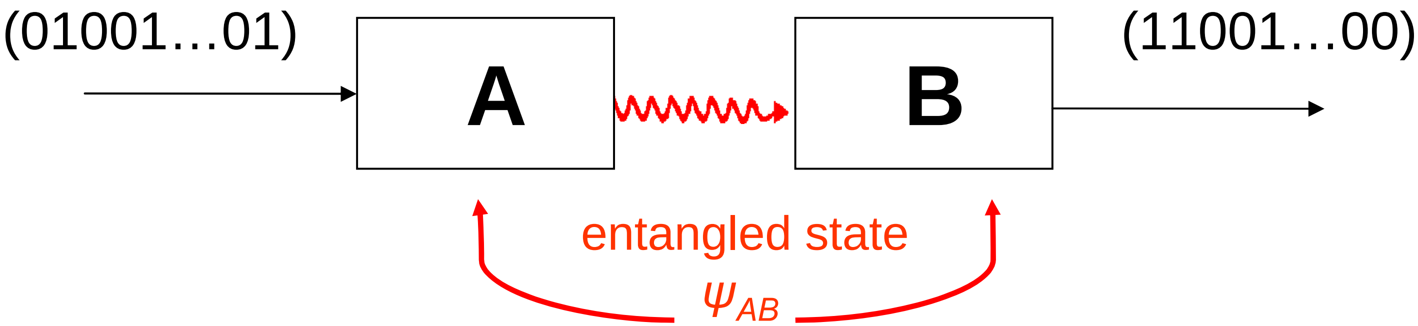
where $\{|x\rangle; x \in \mathcal{X}\}$ is a fixed orthonormal basis.
In the case of *q-c channel* $\rho \to \mathrm{Tr}\rho M_y \,(y \in \mathcal{Y})$ corresponding to measurement of discrete observable (POVM) $M = \{M_y; y \in \mathcal{Y}\}$

$$\mathcal{M}(\rho) = \sum_{y\in\mathcal{Y}} (\mathrm{Tr}\rho M_y)\, |y\rangle\langle y|. \tag{2}$$

Channel performance



classical capacity C



entanglement-assisted classical capacity C_{ea}

(*purification capacity, measurement strength, forward classical communication cost*)

Gain of entanglement-assistance C:C_{ea}

Experimental demonstrations: Kimble et al, quant-ph/0003094, Furusawa et al., quant-ph/0402040

I. MEASUREMENTS: FINITE DIMENSIONS

THE CLASSICAL CAPACITY OF MEASUREMENT

For the *measurement channel* $\mathcal{M}$ HSW-theorem gives

$$C(\mathcal{M}) = \max_{\pi} J(\pi; M),$$

where $\pi = \{\pi_x, \rho_x\}$ are the input *state ensembles* and

$$J(\pi; M) = H\left(P_{\bar{\rho}_\pi}\right) - \sum_x \pi_x H\left(P_{\rho_x}\right)$$

is the *Shannon information* between the input x and the output y. Here $\bar{\rho}_\pi = \sum_x \pi_x \rho_x$, $P_\rho(y) = \mathrm{Tr}\rho M_y$ – the probability distribution of the measurement outcomes and $H(P)$ is the Shannon entropy.

THE ENTANGLEMENT-ASSISTED CLASSICAL CAPACITY OF MEASUREMENT

BSST-theorem gives

$$C_{ea}(\mathcal{M}) = \max_{\rho} I(\rho; \mathcal{M}),$$

where the quantum mutual information:

$$I(\rho; \mathcal{M}) = S(\rho) + S(\mathcal{M}(\rho)) - S(\rho, \mathcal{M}).$$

For the *measurement channel* it equals the *entropy reduction*

$$I(\rho; \mathcal{M}) = S(\rho) - \sum_{y} p_y S(\rho(y|M)), \qquad (3)$$

where $p_y = \mathrm{Tr}\rho M_y$ and $\frac{V_y \rho V_y^*}{p_y} = \rho(y|M)$ are *posterior states* of the measurement ($M_y = V_y^* V_y$). The last term in (3) is *information loss* (Shirokov arXiv:1011.3127, AH arXiv:1103.2615).

APPLICATION: OVERCOMPLETE SYSTEMS

Consider discrete observable $M = \{M_y\}$:

$$M_y = |\psi_y\rangle\langle\psi_y|, \quad \sum_y |\psi_y\rangle\langle\psi_y| = I.$$

The posterior states $\rho\left(y|M\right) = \frac{|\psi_y\rangle\langle\psi_y|}{\langle\psi_y|\psi_y\rangle}$ are *pure*, $S\left(\rho\left(y|M\right)\right) = 0$ and the *information loss is zero*. Thus $I(\rho;\mathcal{M}) = S\left(\rho\right)$ and

$$C_{ea}(\mathcal{M}) = \max_{\rho} S\left(\rho\right) = \log \dim \mathcal{H}.$$

Condition (5) below implies

$$C(\mathcal{M}) < C_{ea}(\mathcal{M}),$$

unless $\{|\psi_y\rangle\}$ *is an orthonormal basis*.

HOW TO COMPUTE?
ENSEMBLE-MEASUREMENT DUALITY

The *dual ensemble* $\pi'_\rho = \left\{ \mathrm{Tr}\rho M_y, \frac{\rho^{1/2} M_y \rho^{1/2}}{\mathrm{Tr}\rho M_y} \right\}$.

Dual *accessible information* $A(\pi'_\rho) = \max_{M''} J(\pi'_\rho, M'')$,

Dall'Arno, D'Ariano and Sacchi, arXiv:1103.1972:

$$C(\mathcal{M}) \equiv \max_{M} J(\pi, M) = \max_{\rho} A(\pi'_\rho).$$

Dual *χ-quantity* $\chi\left(\pi'_\rho\right) = S\left(\sum_y p_y \rho_y\right) - \sum_y p_y S\left(\rho_y\right).$

AH, arXiv:1103.2615:

$$C_{ea}(\mathcal{M}) = \max_\rho \chi\left(\pi'_\rho\right). \tag{4}$$

Information inequality: $A(\pi'_\rho) \leq \chi\left(\pi'_\rho\right)$, equality iff ρ'_y commute.

The equality $C_{ea}(\mathcal{M}) = C(\mathcal{M})$ holds iff

$$\rho^{1/2} M_y \rho M_{y'} \rho^{1/2} = \rho^{1/2} M_{y'} \rho M_y \rho^{1/2} \tag{5}$$

for a density operator ρ maximizing the quantity (4).

APPLICATION: TOMOGRAPHY IN FINITE DIMENSIONS

Let Θ be the unit sphere in $\mathcal{H}, \dim \mathcal{H} = m$, and $\nu(d\theta)$ the uniform distribution on Θ. Then

$$m \int_{\Theta} |\theta\rangle\langle\theta|\nu(d\theta) = I,$$

– a *continuous overcomplete system*, i.e. (unsharp) observable $M(d\theta) = m|\theta\rangle\langle\theta|\nu(d\theta)$ in $\mathcal{H}$ with values in Θ . Information loss is zero and the entanglement-assisted capacity

$$C_{ea}(\mathcal{M}) = \log m.$$

The unassisted classical capacity

$$C(\mathcal{M}) = \log m - \log e \sum_{k=2}^{m} \frac{1}{k}.$$

(arXiv:1103.2615). For $m \to \infty$

$$C(\mathcal{M}) \to \log e \,(1-\gamma),$$

where $\gamma \approx 0.577$ is Euler's constant. At the same time,

$$C_{ea}(\mathcal{M}) = \log m \to \infty.$$

The *gain of entanglement assistance* $C_{ea}/C \approx \log m$.

II. MEASUREMENTS: INFINITE DIMENSIONS, INPUT CONSTRAINT

INFINITE DIMENSIONS: CONSTRAINED CAPACITIES

The input constraint onto transmitted states ρ:

$$\mathrm{Tr}\rho H \leq E, \quad H - \text{energy operator.}$$

The *constrained classical capacity* (for additive channels)

$$C(\Phi, E) = \max_{\rho : \mathrm{Tr}\rho H \leq E} \chi_{\Phi}(\rho), \tag{6}$$

and the *constrained entanglement-assisted classical capacity*

$$C_{ea}(\Phi, E) = \max_{\rho : \mathrm{Tr}\rho H \leq E} I(\rho; \Phi). \tag{7}$$

That gaps in the frequency spectrum reduce the capacity can be understood immediately in terms of the approach used in this section. If there are gaps, the dimension $\mathcal{N}_N$ of the subspace with fixed energy E_N is inevitably reduced, because, in number-theoretic terms, some of the integers are unavailable for partitioning N. Reducing the dimensions of the fixed-energy subspaces obviously reduces the optimum capacities obtained in the preceding three subsections.

The key to our work on wideband capacities is, of course, Holevo's theorem. As we mention in Sec. IV.B, Holevo proved his theorem for a channel that has a finite-dimensional Hilbert space and that uses finite input and output alphabets. By assuming a finite transmission time

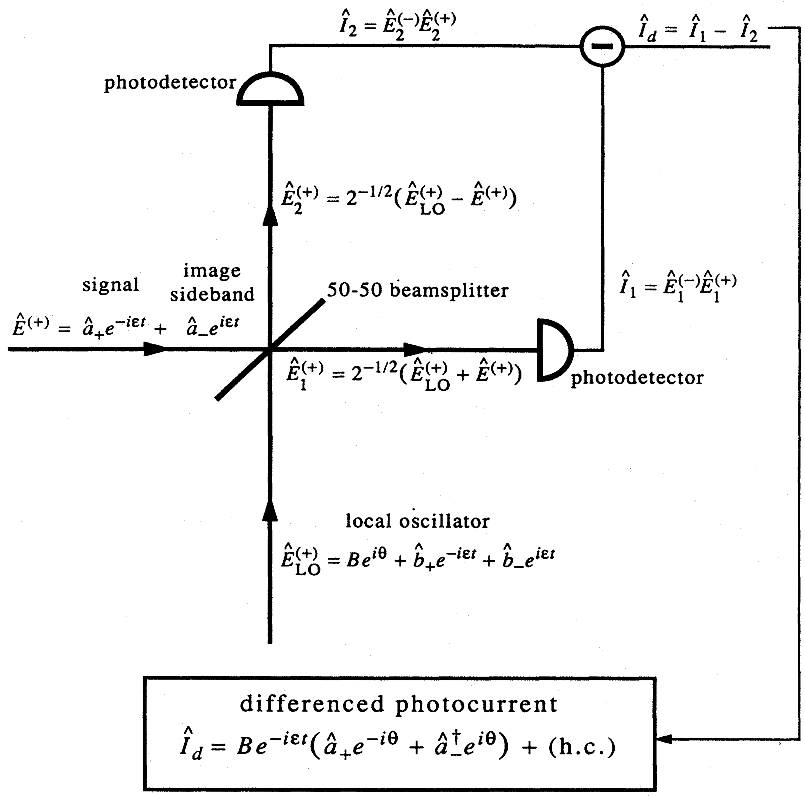


FIG. 4. Balanced heterodyne detection of an optical-frequency signal. The signal to be detected, at frequency $\Omega + \epsilon$, is combined at a 50-50 beam splitter with a much more powerful, coherent beam at frequency Ω. This powerful beam, produced by a laser, is called the local oscillator (LO). The two outputs of the beam splitter are directed onto photodetectors, whose output photocurrents are differenced. The differenced photocurrent $\hat{I}_d$ contains a contribution at the difference frequency ϵ, due to beats between the LO and the signal; it necessarily contains another contribution at the same frequency, due to beats between the LO and the "image sideband," which accompanies the signal into the detector, but which has frequency $\Omega - \epsilon$. The two inputs to the detector are labeled by positive-frequency field operators $\hat{E}^{(+)}$ (signal direction) and $\hat{E}_{LO}^{(+)}$ (LO direction). These field operators have the common optical-frequency oscillation at frequency Ω removed. The two inputs are followed through the detector, and their contributions to the differenced photocurrent are calculated in the limit of a very powerful LO (LO amplitude $B \to \infty$; terms smaller than linear in B are neglected in $\hat{I}_d$). If the differenced photocurrent is filtered to pick out the beat terms at frequency ϵ, the result is detection of the quantity $\hat{a}_+e^{-i\theta} + \hat{a}_-^\dagger e^{i\theta}$, where $\hat{a}_+$ is the modal annihilation operator for the signal mode, $\hat{a}_-^\dagger$ is the modal creation operator for the image sideband, and θ is the LO phase. Since the image sideband is in vacuum, heterodyne detection realizes a measurement of $\hat{a}_+$—i.e., a simultaneous measurement of both quadrature components of the signal (the real and imaginary parts of $\hat{a}_+$). The vacuum noise introduced by the image sideband represents the noise that must be added when both quadrature components are measured simultaneously. This version of heterodyne detection is called balanced because it uses the symmetry between the two outputs of the beam splitter to eliminate vacuum and excess noise that accompanies the LO at frequencies $\Omega \pm \epsilon$ (annihilation operators $\hat{b}_\pm$). Homodyne detection occurs if the differenced photocurrent is filtered to pick out the dc component ($\epsilon = 0$); then the signal and image sideband are the same, so homodyne detection realizes a measurement of the signal quadrature component $\frac{1}{2}(\hat{a}e^{-i\theta} + \hat{a}^\dagger e^{i\theta})$, which depends on the LO phase.

APPLICATION: HETERODYNE MEASUREMENT

One Bosonic mode Q, P, the unsharp observable given by POVM

$$M_{het}(d^2z) = |z\rangle\langle z|\frac{d^2z}{\pi}; \quad z \in \mathbb{C}.$$

This describes *approximate joint measurement* of Q, P and in quantum optics is (approximately) realized by *heterodyning*. The corresponding q-c channel $\mathcal{M}_{het}$ transforms a density operator into the probability distribution

$$\rho \to p_\rho(z) = \langle z|\rho|z\rangle\frac{d^2z}{\pi},$$

with the density $\langle z|\rho|z\rangle$ equal to the Husimi function. The posterior states are the coherent states $\rho\left(z|\mathcal{M}_{het}\right) = |z\rangle\langle z|$ which are pure and have zero entropy. Thus information loss is zero and

$$I(\rho; \mathcal{M}_{het}) = S\left(\rho\right).$$

ENTANGLEMENT-ASSISTED CAPACITY

Let ρ_E be the thermal Gaussian state with number of quanta $\mathrm{Tr}\rho_E a^\dagger a = E$ which maximizes the quantum entropy under the constraint

$$\mathrm{Tr}\rho a^\dagger a \leq E, \tag{8}$$

namely

$$\max_{(8)} S(\rho) = S(\rho_E) = (E+1)\log(E+1) - E\log E \equiv g(E).$$

The formula (7) gives *entanglement-assisted classical capacity* of channel $\mathcal{M}_{het}$ with the constraint (8):

$$C_{ea}(\mathcal{M}_{het}; E) = \max_{(8)} I(\rho; \mathcal{M}_{het}) = g(E).$$

UNASSISTED CLASSICAL CAPACITY

The *classical capacity* (arXiv:1103.2615)

$$C(\mathcal{M}_{het}; E) = \log(E+1),$$

which is always less than

$$C_{ea}(\mathcal{M}_{het}; E) = g(E) = \log(E+1) + \log\left(1+\frac{1}{E}\right)^{E}.$$

The gain $C_{ea}/C \approx -\log E,\ E \to +0$ (weak signal).

APPLICATION: HOMODYNE MEASUREMENT

One Bosonic mode Q, P, the sharp observable given by spectral measure

$$M_{hom}(dx) = |x\rangle\langle x|dx; \qquad x \in \mathbb{R}.$$

This describes *exact measurement* of Q and in quantum optics is (approximately) realized by *homodyning*. The corresponding q-c channel $\mathcal{M}_{hom}$ takes a density operator into the probability distribution

$$\rho \to p_\rho(x) = \langle x|\rho|x\rangle dx.$$

The posterior states are again pure and have zero entropy. Thus $I(\rho; \mathcal{M}_{hom}) = S(\rho)$. The formula (7) gives *entanglement-assisted classical capacity* of channel $\mathcal{M}_{hom}$ with the constraint (8):

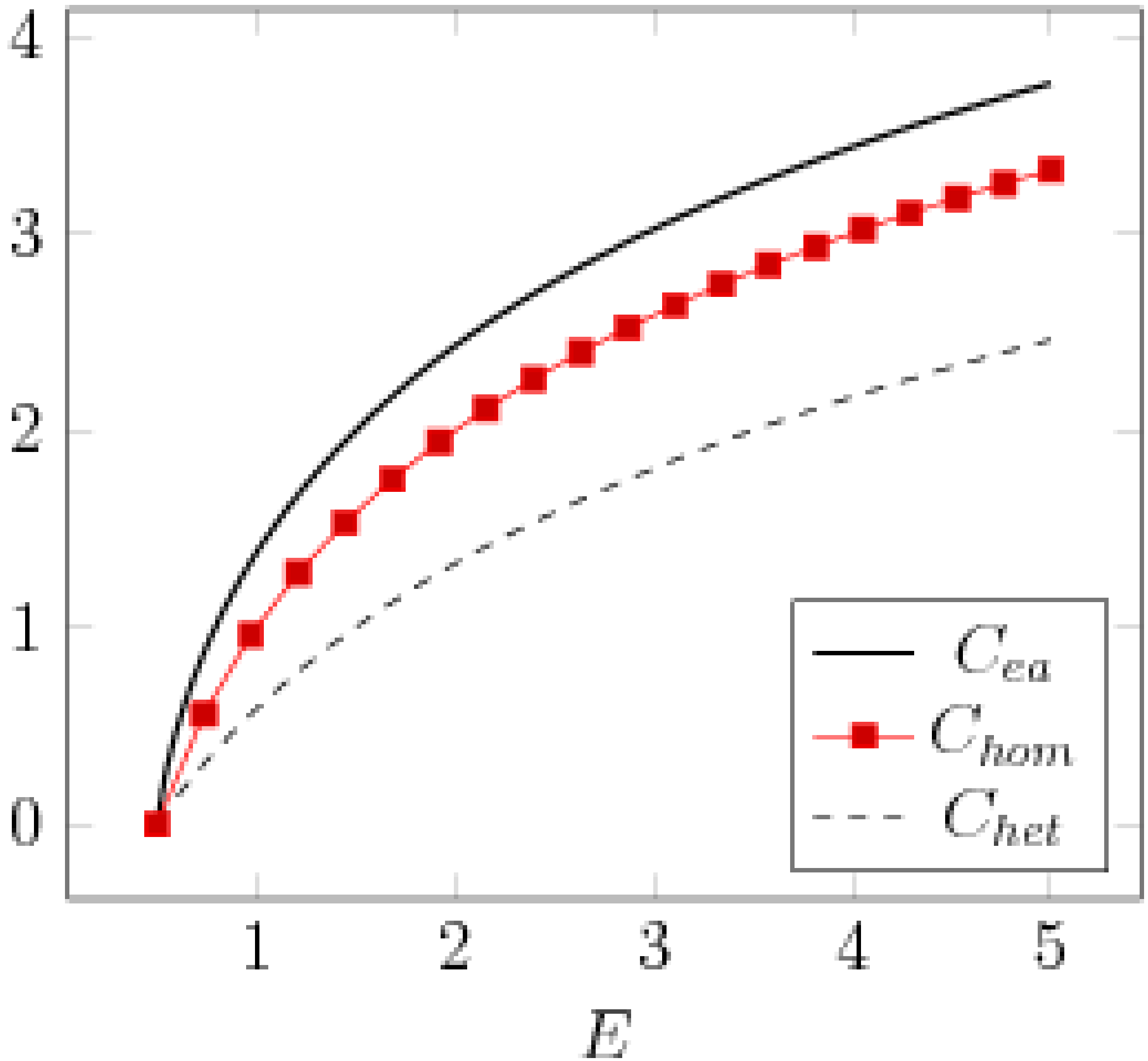
$$C_{ea}(\mathcal{M}_{hom}; E) = (E+1)\log(E+1) - E\log E \equiv g(E)$$

(same as heterodyning channel).

The *unassisted classical capacity* (M.J.W. Hall, Information exclusion principle)

$$C(\mathcal{M}_{hom}; E) = \log(2E+1),$$

The gain $C_{ea}/C \approx -\frac{1}{2}\log E,\ E \to +0$ (weak signal).



4
3
2
1
0
1
2
3
4
5
E
C_{ea}
C_{hom}
C_{het}

III. PREPARATION CHANNELS

PREPARATIONS: FINITE DIMENSIONS, NO CONSTRAINT

Shirokov arXiv:1105.1040, Shirokov, AH arXiv:1210.6926:
Let Φ be finite-dimensional channel, then

- If the channel Φ is c-q (preparation) then $C_{ea}(\Phi) = C(\Phi)$;
- If $C_{ea}(\Phi) = C(\Phi)$ then the channel Φ is "essentially" c-q channel.

PREPARATIONS: ONE-MODE COMMUNICATION

Caves and Drummond, p.511:

"A classical current source radiates a coherent state of the electromagnetic field (Glauber, 1963b, 1965). The phased dipoles in an ideal laser operated far above threshold approximate very closely a classical current source, except for a slow diffusion in phase; thus, for times short compared to the phase-diffusion time, a laser produces a close approximation to a coherent state."

Typically, the coherent signal is degraded by additive Gaussian noise resulting in a displaced thermal state, which models the effects of thermal noise, linear amplifier/attenuator noise, and inefficient photodetection.

CASE 1: TWO-QUADRATURES SIGNAL

The typical case with $C(E) = C_{ea}(E)$:

$$m = (m_q, m_p) \to \rho_m = D(m)\rho_0 D(m)^\dagger,$$

where $D(m) = \exp i(m_p q - m_q p)$ is displacement operator, ρ_0 is the Gaussian thermal state with the mean photon number N. This channel can be considered as transmission of the *classical signal* $m = (m_q, m_p)$ plus *additive quantum Gaussian noise* q, p with average number of quanta N. The *minimal noise* corresponds to $N = 0$. Input constraint

$$\frac{1}{2}\int (m_q^2 + m_p^2) p(m) d^2 m \leq E.$$

THE CLASSICAL CAPACITY

The maximum entropy principle gives the *constrained capacity*

$$C(E) = g(N+E) - g(N),$$

where $g(N) \equiv (N+1)\log(N+1) - N\log N = S(\rho_m)$, with the optimal Gaussian distribution

$$p(m) = \frac{1}{2\pi E}\exp\left(-\frac{\left(m_q^2 + m_p^2\right)}{2E}\right).$$

In particular, for the *quantum-limited* channel ($N = 0$),

$$C(E) = g(E).$$

In this case (AH arXiv:1211.4774)

$$C_{ea}(E) = C(E) = g(E).$$

EMBEDDING INTO QUANTUM CHANNEL

To compute *entanglement-assisted* capacity, we represent this channel as quantum Gaussian channel Φ. Since the *input* $m = (m_q, m_p)$ is two-dimensional classical, one has to use two Bosonic modes $q_1, p_{1,}, q_2, p_2$ to describe it quantum-mechanically, e.g. $m_q = q_1, m_p = q_2$. The *environment* is one mode q, p in the Gaussian state ρ_0, so the *output* is given by the equations

$$\begin{aligned} q' &= q + q_1 = q + m_q; \\ p' &= p + q_2 = p + m_p, \end{aligned} \tag{9}$$

DYNAMICS OF ENVIRONMENT

The equations for the environment modes are

$$\begin{aligned} q_1' &= q_1, \\ p_1' &= p_1 - p - q_2/2, \\ q_2' &= q_2, \\ p_2' &= p_2 + q + q_1/2. \end{aligned}$$

If $N = 0$ so that ρ_0 is pure, these equations describe the *complementary channel* $\tilde{\Phi}$.

THE ENTANGLEMENT-ASSISTED CLASSICAL CAPACITY. I

Having realized the c-q channel as a quantum one, it makes sense to speak of *entanglement-assisted capacity* of this channel:

$$C_{ea}(E) = \max_{\rho_{12} \in \mathfrak{S}_E} I(\rho_{12}, \Phi),$$

where $I(\rho_{12}, \Phi) = S(\rho_{12}) + S(\Phi[\rho_{12}]) - S(\tilde{\Phi}[\rho_{12}])$ is the quantum mutual information and $\mathfrak{S}_E = \left\{ \rho_{12} : \mathrm{Tr}\rho_{12} \left(\frac{q_1^2 + q_2^2}{2} \right) \leq E \right\}$. Assuming the minimal noise $N = 0$, *equations for the complementary channel imply* $S(\tilde{\Phi}[\rho_{12}]) \geq S(\rho_{12})$ whence

$$C_{ea}(E) \leq \max_{\rho_{12} \in \mathfrak{S}_E} S(\Phi[\rho_{12}]).$$

THE ENTANGLEMENT-ASSISTED CLASSICAL CAPACITY. II

The output states are

$$\Phi[\mathfrak{S}_E] = \{\bar{\rho}_p : p \in \mathcal{P}_E\}\,,$$

as can be seen from the equations of the channel (9) and the identification of the probability density $p(m_q\,, m_p)$ with that of observables q_1, q_2 in the state ρ_{12}. Thus

$$\max_{\rho_{12}\in\mathfrak{S}_E} S(\Phi[\rho_{12}]) = S(\bar{\rho}) = g(E)$$

and hence

$$C_{ea}(E) = C(E) = g(E).$$

CASE 2: ONE-QUADRATURE SIGNAL

A case with $C(E) < C_{ea}(E)$. Let $m \in \mathbf{R}$ be a real one-dimensional signal and the channel is

$$m \to \mathrm{e}^{-imp}\rho_0\mathrm{e}^{imp},$$

where ρ_0 is the squeezed vacuum with variances $\mathrm{D}q = \sigma^2, \mathrm{D}p = \frac{1}{4\sigma^2}$ and zero covariance between q and p. The channel $m \to \rho_m$ can be considered as transmission of the *classical signal* m with the *additive noise* arising from the q -quadrature of quantum Gaussian mode q, p.

THE CLASSICAL CAPACITY

The constraint on the input probability distribution $p(m)$:

$$\int m^2 \, p(m) dm \leq E, \tag{10}$$

where E is a positive constant. As the component p is not affected by the signal, from information-theoretic point this channel is equivalent to the classical additive Gaussian noise channel $m \to m + q$, and its capacity under the constraint (10) is given by the Shannon formula

$$C(E) = \frac{1}{2} \log \left(1 + r\right),$$

where $r = E/\sigma^2$ is the *signal-to-noise ratio* (SNR).

EMBEDDING INTO QUANTUM CHANNEL

Introducing the *input* mode q_1, p_1, so that $m = q_1$, with the *environment* mode q, p in the state ρ_0, the *output* is given by the equations

$$\begin{aligned} q_1' &= q_1 + q; \\ p_1' &= \qquad p. \end{aligned} \tag{11}$$

The equations for the environment mode describing the *complementary channel* $\tilde{\Phi}$ are

$$\begin{aligned} q' &= q_1, \\ p' &= p_1 - p, \end{aligned} \tag{12}$$

and the set of equations (11), (12) describes the canonical transformation of the composite system = system+environment.

THE ENTANGLEMENT-ASSISTED CAPACITY

The *constrained classical entanglement-assisted capacity* of this channel is

$$C_{ea}(E) = \max_{\rho_1 \in \mathfrak{S}_E^{(1)}} I(\rho_1, \Phi),$$

where $\mathfrak{S}_E^{(1)} = \left\{ \rho_1 : \mathrm{Tr}\rho_1 q_1^2 \leq E \right\}$.

To compute $C_{ea}(E)$ we consider the values of $I(\rho_A, \Phi)$ for centered Gaussian states $\rho_A = \rho_1$ with covariance matrices

$$\alpha_1 = \begin{bmatrix} E & 0 \\ 0 & E_1 \end{bmatrix},$$

satisfying the uncertainty relation $EE_1 \geq \frac{1}{4}$ and belonging to the set $\mathfrak{S}_E^{(1)}$ with the equality.

THE GAIN OF ENTANGLEMENT-ASSISTANCE

Computation of the three entropies in the limit $E_1 \to \infty$ gives

$$C_{ea}(E) = g\left(\sqrt{\frac{E}{4\sigma^2} + \frac{1}{4}} - \frac{1}{2}\right) = g\left(\frac{\sqrt{1+r} - 1}{2}\right).$$

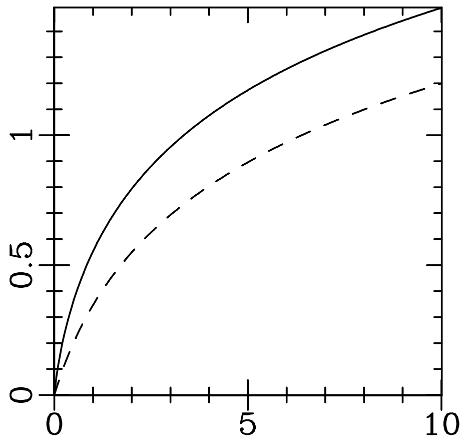
Comparing this with

$$C(E) = \frac{1}{2}\log\left(1 + r\right),$$

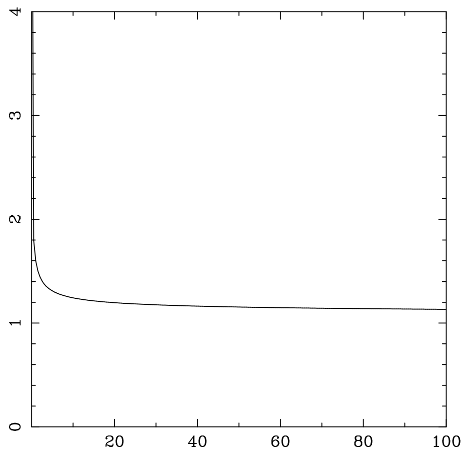
one has $C_{ea}(E) > C(E)$ for $E > 0$, with the *gain of entanglement-assistance*

$$C_{ea}(E)/C(E) \sim -\frac{1}{2}\log r, \quad r \to 0,$$

and $C_{ea}(E)/C(E) \to 1$, as $r \to \infty$.



$C_{ea}(E)$ *vs* $C(E)$.



The gain $C_{ea}(E)/C(E)$.

CONCLUSIONS

- The gain of entanglement-assistance $C_{ea}/C > 1$ generically for measurement (q-c) channels with *unsharp* observables;

- For *pure* measurements (pure posterior states) the information loss in the entanglement-assisted protocol is zero, resulting in maximal gain for weak signals (typically $C_{ea}/C \sim -\log \mathtt{SNR}$);

- For c-q channels (state preparations) there is essentially *no gain* of entanglement-assistance, $C_{ea}/C = 1$. But the *input constraints* (e.g. energy constraint in continuous variable systems) can restore the gain for c-q channels.